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In $\triangle ABC$ the following relationship holds:

$$\sin A \sin B \sin C \leq \cos \frac{A}{2} \cos \frac{B}{2} \cos \frac{C}{2}$$

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We use:

$$\sin A \sin B \sin C = \frac{sr}{2R^2}; \cos \frac{A}{2} \cos \frac{B}{2} \cos \frac{C}{2} = \frac{s}{4R}$$

We must prove that:

$$\frac{sr}{2R^2} \leq \frac{s}{4R} \Leftrightarrow \frac{r}{2R^2} \leq \frac{1}{4R} \Leftrightarrow 2r \leq R \quad (\text{Euler})$$

Equality holds for $A = B = C$.