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In $\triangle ABC$ the following relationship holds:

$$\sqrt{a \sin A} + \sqrt{b \sin B} + \sqrt{c \sin C} \leq \frac{3\sqrt{6R}}{2}$$

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$$\begin{aligned}\sqrt{a \sin A} + \sqrt{b \sin B} + \sqrt{c \sin C} &= \sum_{cyc} \sqrt{a \sin A} = \\ &= \sum_{cyc} \sqrt{a \cdot \frac{a}{2R}} = \sum_{cyc} \frac{a}{\sqrt{2R}} = \frac{2s}{\sqrt{2R}} = \frac{s\sqrt{2}}{\sqrt{R}}\end{aligned}$$

Remains to prove:

$$\begin{aligned}\frac{s\sqrt{2}}{\sqrt{R}} \leq \frac{3\sqrt{6R}}{2} &\Leftrightarrow \frac{2s^2}{R} \leq \frac{9 \cdot 6R}{4} \\ 8s^2 \leq 54R^2 &\Leftrightarrow s^2 \leq \frac{27R^2}{4} \Leftrightarrow s \leq \frac{3\sqrt{3}R}{2} \quad (\text{MITRINOVIC})\end{aligned}$$

Equality holds for $a = b = c$.