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In ΔABC the following relationship holds:

$$\sqrt{a\sin\frac{A}{2}}+\sqrt{b\sin\frac{B}{2}}+\sqrt{c\sin\frac{C}{2}}\le\frac{3\sqrt[4]{12R^2}}{2}$$

Proposed by Nguyen Hung Cuong – Vietnam

Solution by Daniel Sitaru – Romania

$$\sqrt{a\sin\frac{A}{2}}+\sqrt{b\sin\frac{B}{2}}+\sqrt{c\sin\frac{C}{2}}\overset{CBS}{\le}\sqrt{\sum_{cyc}a\cdot\sum_{cyc}\sin\frac{A}{2}}\overset{JENSEN}{\ge}\sqrt{2s\cdot 3\sin\left(\frac{A+B+C}{6}\right)}=$$

$$=\sqrt{6s\cdot\sin\frac{\pi}{6}}=\sqrt{6s\cdot\frac{1}{2}}=\sqrt{3s}$$

Remains to prove:

$$\sqrt{3s}\le\frac{3\cdot\sqrt[4]{12R^2}}{2}\Leftrightarrow 3s\le\frac{9\sqrt{12R^2}}{4}\Leftrightarrow 4s\le 3\sqrt{12R^2}\Leftrightarrow s\le\frac{3\cdot 2\sqrt{3}R}{4}\Leftrightarrow s\le\frac{3\sqrt{3}R}{2}$$

(MITRINOVIC)

Equality holds for $a=b=c$.