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**In $\Delta ABC$ the following relationship holds:**

$$\sqrt{a\sin^3 A}+\sqrt{b\sin^3 B}+\sqrt{c\sin^3 C}\le\frac{9\sqrt{2R}}{4}$$

*Proposed by Nguyen Hung Cuong – Vietnam*

*Solution by Daniel Sitaru – Romania*

$$\sqrt{a\sin^3 A}+\sqrt{b\sin^3 B}+\sqrt{c\sin^3 C}=\sum_{cyc}\sqrt{a\sin^3 A}=$$

$$=\sum_{cyc}\sqrt{2R\sin^4 A}=\sqrt{2R}\sum_{cyc}\sin^2 A=\sqrt{2R}\sum_{cyc}\frac{a^2}{4R^2}=$$

$$=\frac{\sqrt{2R}}{4R^2}\cdot\sum_{cyc}a^2=\frac{\sqrt{2R}}{4R^2}\cdot 2(s^2-r^2-4Rr)\le$$

$$\overset{GERRETSEN}{\le}\frac{\sqrt{2R}}{2R^2}(4R^2+4Rr+3r^2-r^2-4Rr)=$$

$$=\frac{\sqrt{2R}}{2R^2}(4R^2+2r^2)\overset{EULER}{\le}\frac{\sqrt{2R}}{2R^2}\left(4R^2+2\cdot\frac{R^2}{4}\right)=\frac{\sqrt{2R}}{2R^2}\cdot\frac{18R^2}{4}=\frac{9\sqrt{2R}}{4}$$

Equality holds for $a=b=c$.