ROMANIAN MATHEMATICAL MAGAZINE

In ΔABC the following relationship holds:

$$\sqrt{a}\sin A+\sqrt{b}\sin B+\sqrt{c}\sin C\le\frac{3}{2}\sqrt[4]{27R^2}$$

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Solution by Daniel Sitaru – Romania

$$\sqrt{a}\sin A+\sqrt{b}\sin B+\sqrt{c}\sin C\overset{CBS}{\le}\sqrt{(a+b+c)(\sin^2 A+\sin^2 B+\sin^2 C)}=$$

$$=\sqrt{2s\cdot\frac{s^2-r^2-4Rr}{2R^2}}\overset{GERRETSEN}{\le}\sqrt{\frac{s}{R^2}(4R^2+4Rr+3r^2-r^2-4Rr)}\overset{MITRINOVIC}{\le}$$

$$\le\sqrt{\frac{3\sqrt{3}}{2}R\cdot\frac{1}{R^2}\cdot(4R^2+2r^2)}\overset{EULER}{\le}\sqrt{\frac{3\sqrt{3}}{2R}\left(4R^2+2\cdot\frac{R^2}{4}\right)}=\sqrt{\frac{3\sqrt{3}}{2R}\cdot\frac{9R^2}{2}}=$$

$$=\frac{3}{2}\sqrt{3\sqrt{3}R}=\frac{3}{2}\sqrt[4]{27R^2}$$

Equality holds for $a=b=c$.