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In $\triangle ABC$ the following relationship holds:

$$\frac{\sin A + \sin B}{\cos \frac{C}{2}} + \frac{\sin B + \sin C}{\cos \frac{A}{2}} + \frac{\sin C + \sin A}{\cos \frac{B}{2}} \leq 6$$

Proposed by Nguyen Hung Cuong – Vietnam

Solution by Daniel Sitaru – Romania

$$\begin{aligned} \sum_{cyc} \frac{\sin A + \sin B}{\cos \frac{C}{2}} &= \sum_{cyc} \frac{2 \sin \frac{A+B}{2} \cos \frac{A-B}{2}}{\cos \frac{C}{2}} = \\ &= 2 \sum_{cyc} \frac{\sin \frac{\pi - C}{2} \cos \frac{A-B}{2}}{\cos \frac{C}{2}} = 2 \sum_{cyc} \frac{\cos \frac{C}{2} \cos \frac{A-B}{2}}{\cos \frac{C}{2}} = 2 \sum_{cyc} \cos \frac{A-B}{2} \stackrel{JENSEN}{\leq} \\ &\leq 2 \cdot 3 \cdot \cos \left(\frac{\frac{A-B}{2} + \frac{B-C}{2} + \frac{C-A}{2}}{3} \right) = 6 \cos \left(\frac{0}{3} \right) = 6 \cos 0 = 6 \cdot 1 = 6 \end{aligned}$$

Equality holds for $A = B = C$.