ROMANIAN MATHEMATICAL MAGAZINE

In acute ΔABC the following relationship holds:

$$\sum_{cyc}\frac{\cot A}{\cot B}\ge\sum_{cyc}\frac{\sin^2 A}{\sin^2 B}$$

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WLOG: $a\le b\le c\Rightarrow\begin{cases}b-a\ge b-c\ge a-c\\ \cot A\le\cot B\le\cot C\end{cases}$

$$\sum_{cyc}(b-a)\cot A\overset{CEBYSHEV}{\ge}\frac{1}{3}\sum_{cyc}(b-a)\cdot\sum_{cyc}\cot A=0\Leftrightarrow\sum_{cyc}(b-a)\cot A\ge 0$$

$$\sum_{cyc}(b+a)(b-a)\cot A\ge 0\Rightarrow\sum_{cyc}(b^2-a^2)\cot A\ge 0$$

$$\Rightarrow\sum_{cyc}(b^2-a^2)\cdot\frac{b^2+c^2-a^2}{4F}\ge 0\Rightarrow\sum_{cyc}(b^2-a^2)(b^2+c^2-a^2)\ge 0$$

$$\sum_{cyc}(b^4+b^2c^2-b^2a^2-a^2b^2-a^2c^2+a^4)\ge 0$$

$$\sum_{cyc}\left(a^2(b^2+c^2-a^2)-b^2(a^2+c^2-b^2)\right)\ge 0$$

$$\sum_{cyc}\frac{b^2+c^2-a^2}{a^2+c^2-b^2}\ge\sum_{cyc}\frac{b^2}{a^2}\Rightarrow\sum_{cyc}\frac{\cot A}{\cot B}\ge\sum_{cyc}\frac{\sin^2 A}{\sin^2 B}$$

Equality holds for $a=b=c$.