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In acute $\triangle ABC$ the following relationship holds:

$$\frac{a \cos A}{HA} + \frac{b \cos B}{HB} + \frac{c \cos C}{HC} \leq \frac{3\sqrt{3}}{2}$$

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Solution by Daniel Sitaru – Romania

$$\begin{aligned} \frac{a \cos A}{HA} + \frac{b \cos B}{HB} + \frac{c \cos C}{HC} &= \sum_{cyc} \frac{a \cos A}{HA} = \sum_{cyc} \frac{2R \sin A \cdot \cos A}{2R \cos A} = \sum_{cyc} \sin A \stackrel{JENSEN}{\leq} \\ &\leq 3 \sin \left(\frac{A+B+C}{3} \right) = 3 \sin \frac{\pi}{3} = 3 \cdot \frac{\sqrt{3}}{2} = \frac{3\sqrt{3}}{2} \end{aligned}$$

Equality holds for $a = b = c$.