ROMANIAN MATHEMATICAL MAGAZINE

In ΔABC the following relationship holds:

$$\frac{GA^2}{a^2}+\frac{GB^2}{b^2}+\frac{GC^2}{c^2}\geq\frac{3}{2}$$

Proposed by Nguyen Hung Cuong – Vietnam

Solution by Daniel Sitaru – Romania

$$\frac{GA^2}{a^2}+\frac{GB^2}{b^2}+\frac{GC^2}{c^2}=\sum_{cyc}\frac{GA^2}{a^2}=\frac{2}{3}\sum_{cyc}\frac{m_a^2}{a^2}=$$

$$=\frac{2}{3}\sum_{cyc}\frac{2(b^2+c^2)-a^2}{4a^2}=\frac{1}{6}\sum_{cyc}\left(\frac{2b^2}{a^2}+\frac{2c^2}{a^2}\right)-\frac{1}{6}\sum_{cyc}\frac{a^2}{a^2}=$$

$$=\frac{2}{6}\left(\left(\frac{b^2}{a^2}+\frac{a^2}{b^2}\right)+\left(\frac{c^2}{b^2}+\frac{b^2}{c^2}\right)+\left(\frac{a^2}{c^2}+\frac{c^2}{a^2}\right)\right)-\frac{1}{6}\cdot 3\geq$$

$$\geq\frac{1}{3}(2+2+2)-\frac{1}{2}=\frac{6}{3}-\frac{1}{2}=2-\frac{1}{2}=\frac{3}{2}$$

Equality holds for $a=b=c$.