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In acute $\triangle ABC$ the following relationship holds:

$$\frac{HA \sin A}{a} + \frac{HB \sin B}{b} + \frac{HC \sin C}{c} \leq \frac{3}{2}$$

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Solution by Daniel Sitaru – Romania

$$\begin{aligned} \frac{HA \sin A}{a} + \frac{HB \sin B}{b} + \frac{HC \sin C}{c} &= \sum_{cyc} \frac{HA \sin A}{a} = \sum_{cyc} \frac{2R \cos A \sin A}{2R \sin A} = \\ &= \sum_{cyc} \cos A \stackrel{JENSEN}{\leq} 3 \cos \left(\frac{A+B+C}{3} \right) = 3 \cos \frac{\pi}{3} = \frac{3}{2} \end{aligned}$$

Equality holds for $a = b = c$.