

ROMANIAN MATHEMATICAL MAGAZINE

In acute $\triangle ABC$ the following relationship holds:

$$\frac{a}{HA \sin A} + \frac{b}{HB \sin B} + \frac{c}{HC \sin C} \geq 6$$

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Solution by Daniel Sitaru – Romania

$$\begin{aligned} \frac{a}{HA \sin A} + \frac{b}{HB \sin B} + \frac{c}{HC \sin C} &= \sum_{cyc} \frac{a}{HA \sin A} = \\ &= \sum_{cyc} \frac{2R \sin A}{2R \cos A \cdot \sin A} = \sum_{cyc} \frac{1}{\cos A} \stackrel{JENSEN}{\geq} 3 \cdot \frac{1}{\cos\left(\frac{A+B+C}{3}\right)} = \frac{3}{\cos \frac{\pi}{3}} = \frac{3}{\frac{1}{2}} = 6 \end{aligned}$$

Equality holds for $a = b = c$.