ROMANIAN MATHEMATICAL MAGAZINE

In acute ΔABC the following relationship holds:

$$\frac{HA}{a}+\frac{HB}{b}+\frac{HC}{c}\geq\sqrt{3}$$

Proposed by Nguyen Hung Cuong – Vietnam

Solution by Daniel Sitaru – Romania

$$\frac{HA}{a}+\frac{HB}{b}+\frac{HC}{c}=\sum_{cyc}\frac{HA}{a}=\sum_{cyc}\frac{2R\cos A}{2R\sin A}=\sum_{cyc}\frac{b^2+c^2-a^2}{2bc\sin A}=\sum_{cyc}\frac{b^2+c^2-a^2}{4F}=$$

$$=\frac{1}{4F}\left(\sum_{cyc}b^2+\sum_{cyc}c^2-\sum_{cyc}a^2\right)=\frac{1}{4F}\sum_{cyc}a^2=$$

$$=\frac{1}{4F}(a^2+b^2+c^2)\overset{IONESCU-WEITZENBOCK}{\geq}\frac{1}{4F}\cdot 4\sqrt{3}F=\sqrt{3}$$

Equality holds for $a=b=c$.