ROMANIAN MATHEMATICAL MAGAZINE

In acute ΔABC the following relationship holds:

$$\frac{HA}{a\cos A}+\frac{HB}{b\cos B}+\frac{HC}{c\cos C}\ge 2\sqrt{3}$$

Proposed by Nguyen Hung Cuong – Vietnam

Solution by Daniel Sitaru – Romania

$$\frac{HA}{a\cos A}+\frac{HB}{b\cos B}+\frac{HC}{c\cos C}=\sum_{cyc}\frac{HA}{a\cos A}=$$

$$=\sum_{cyc}\frac{2R\cos A}{a\cos A}=2R\sum_{cyc}\frac{1}{a}=\sum_{cyc}\frac{2R}{2R\sin A}=$$

$$=\sum_{cyc}\frac{1}{\sin A}\overset{JENSEN}{\ge}3\cdot\frac{1}{\sin\left(\frac{A+B+C}{3}\right)}=\frac{3}{\sin\frac{\pi}{3}}=\frac{3}{\frac{\sqrt{3}}{2}}=\frac{6}{\sqrt{3}}=\frac{6\sqrt{3}}{3}=2\sqrt{3}$$

Equality holds for $a=b=c$.