ROMANIAN MATHEMATICAL MAGAZINE

In acute ΔABC the following relationship holds:

$$\frac{abc}{HA\cdot HB\cdot HC}\geq 3\sqrt{3}$$

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Solution by Daniel Sitaru – Romania

$$\frac{abc}{HA\cdot HB\cdot HC}\geq 3\sqrt{3}\Leftrightarrow \frac{2R\sin A\cdot 2R\sin B\cdot 2R\sin C}{2R\cos A\cdot 2R\cos B\cdot 2R\cos C}\geq 3\sqrt{3}$$

$$\frac{\sin A}{\cos A}\cdot\frac{\sin B}{\cos B}\cdot\frac{\sin C}{\cos C}\geq 3\sqrt{3}$$

$$\tan A\cdot\tan B\cdot\tan C\geq 3\sqrt{3}$$

$$\tan A+\tan B+\tan C\geq 3\sqrt{3}\quad \text{(to prove)}$$

$$\tan A+\tan B+\tan C\overset{JENSEN}{\geq}3\tan\left(\frac{A+B+C}{3}\right)=3\tan\left(\frac{\pi}{3}\right)=3\sqrt{3}$$

Equality holds for $A=B=C$.