

ROMANIAN MATHEMATICAL MAGAZINE

In any acute ΔABC the following relationship holds :

$$\sum_{\text{cyc}} \sqrt{a \cos A} \leq \frac{3 \sqrt[4]{12R^2}}{2}$$

Proposed by Nguyen Hung Cuong-Vietnam

Solution by Soumava Chakraborty-Kolkata-India

$$\begin{aligned} \sum_{\text{cyc}} \sqrt{a \cos A} &\stackrel{\text{CBS}}{\leq} \sqrt{3} \cdot \sqrt{\sum_{\text{cyc}} (a \cos A)} = \sqrt{3R} \cdot \sqrt{\sum_{\text{cyc}} \sin 2A} = \sqrt{3R} \cdot \sqrt{4 \cdot \frac{4Rrs}{8R^3}} \\ &= \sqrt{\frac{6rs}{R}} \stackrel{\text{Euler and Mitrinovic}}{\leq} \sqrt{\frac{3R}{2R} \cdot 3\sqrt{3} \cdot R} = \frac{3}{2} \cdot \sqrt{2\sqrt{3} \cdot R} = \frac{3}{2} \cdot \sqrt{\sqrt{12R^2}} \\ \therefore \sum_{\text{cyc}} \sqrt{a \cos A} &\leq \frac{3 \cdot \sqrt[4]{12R^2}}{2} \quad \forall \text{ acute } \Delta ABC, " = " \text{ iff } \Delta ABC \text{ is equilateral (QED)} \end{aligned}$$