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In ΔABC the following relationship holds:

$$\sqrt{\sum_{cyc} \frac{1}{(r_a + r_b)\sin(A)\sin(B)}} \geq \frac{3\sqrt{R}}{s}$$

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Solution by Mirsadix Muzefferov-Azerbaijan

$$\frac{1}{(r_a + r_b)\sin(A)\sin(B)} = \frac{4R^2}{(r_a + r_b)ab} = \frac{4R^2.c^2}{(r_a + r_b)abc.c} =$$

$$= \frac{4R^2.c^2}{(r_a + r_b)4RF.c} = \frac{R.c^2}{(r_a + r_b)F.c}$$

$$\sum_{cyc} \frac{R.c^2}{(r_a + r_b)F.c} \overset{Bergstrom}{\geq} \frac{R}{F}\cdot\frac{\left(\sum_{cyc} c\right)^2}{\sum_{cyc}(r_a + r_b).c} = \frac{R}{F}\cdot\frac{4s^2}{4s(R + r)} =$$

$$= \frac{R}{sr}\cdot\frac{4s^2}{4s(R + r)} = \frac{R}{Rr + r^2} \geq \frac{R}{\frac{s^2}{9}} = \frac{9R}{s^2} \text{ because: } \therefore Rr + r^2 \leq \frac{2s^2}{27} + \frac{s^2}{27} = \frac{s^2}{9}$$

$$\sqrt{\sum_{cyc} \frac{1}{(r_a + r_b)\sin(A)\sin(B)}} \geq \sqrt{\frac{9R}{s^2}} = \frac{3\sqrt{R}}{s}$$

Equality holds for: $a = b = c$.