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In any Δ ABC the following relationship holds :

$$\sum_{cyc}\left(\tan\frac{A}{2}\cdot\sqrt{\frac{1}{3}\left(\tan^2\frac{B}{2}+\tan\frac{B}{2}\tan\frac{C}{2}+\tan^2\frac{C}{2}\right)}\right)\geq 1$$

Proposed by Neculai Stanciu, Viorel Ovidiu Ignatescu-Romania

Solution by Soumava Chakraborty-Kolkata-India

$$\text{LHS}\geq\sum_{cyc}\left(\tan\frac{A}{2}\cdot\sqrt{\frac{1}{3}\cdot\frac{3}{4}\cdot\left(\tan\frac{B}{2}+\tan\frac{C}{2}\right)^2}\right)=$$

$$=\frac{1}{2}\sum_{cyc}\left(\tan\frac{A}{2}\left(\tan\frac{B}{2}+\tan\frac{C}{2}\right)\right)=\sum_{cyc}\left(\tan\frac{A}{2}\tan\frac{B}{2}\right)=1$$

$$\therefore\sum_{cyc}\left(\tan\frac{A}{2}\cdot\sqrt{\frac{1}{3}\left(\tan^2\frac{B}{2}+\tan\frac{B}{2}\tan\frac{C}{2}+\tan^2\frac{C}{2}\right)}\right)\geq 1\ \forall\ ABC,$$

$$''=''\text{ iff }\Delta\text{ ABC is equilateral (QED)}$$