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In ΔABC the following relationship holds:

$$\sum_{cyc}\frac{\tan^2\left(\frac{A}{2}\right)}{h_a}\geq\sum_{cyc}\frac{\tan^2\left(\frac{A}{2}\right)}{r_a}$$

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$$In\quad \Delta ABC\ wlog\ a\leq b\leq c;\ h_a\geq h_b\geq h_c;\ r_a\leq r_b\leq r_c$$

$$\tan\left(\frac{A}{2}\right)\leq\tan\left(\frac{B}{2}\right)\leq\tan\left(\frac{C}{2}\right);\ \frac{1}{h_a}\leq\frac{1}{h_b}\leq\frac{1}{h_c};\ \frac{1}{r_a}\geq\frac{1}{r_b}\geq\frac{1}{r_c}$$

$$\sum_{cyc}\frac{\tan^2\left(\frac{A}{2}\right)}{h_a}\overset{Chebyshev}{\underset{\smile}{\geq}}\frac{1}{3}\sum_{cyc}\frac{1}{h_a}.\sum_{cyc}\tan^2\left(\frac{A}{2}\right)=\frac{1}{3r}\sum_{cyc}\tan^2\left(\frac{A}{2}\right)\ (1)$$

$$\sum_{cyc}\frac{\tan^2\left(\frac{A}{2}\right)}{r_a}\overset{Chebyshev}{\underset{\smile}{\leq}}\frac{1}{3}\sum_{cyc}\frac{1}{r_a}.\sum_{cyc}\tan^2\left(\frac{A}{2}\right)=\frac{1}{3r}\sum_{cyc}\tan^2\left(\frac{A}{2}\right)\ (2)$$

$$From\ \ (1)\ and\ (2)\ \ we\ \ obtain..$$

$$LHS\geq\frac{1}{3r}\sum_{cyc}\tan^2\left(\frac{A}{2}\right)\geq RHS\ (Proved)$$

$$Equality\ holds\ iff:\quad a=b=c$$