

ROMANIAN MATHEMATICAL MAGAZINE

In $\triangle ABC$ the following relationship holds:

$$27r \leq \sum_{cyc} r_a \cdot \operatorname{ctg}^2 \left(\frac{A}{2} \right) \leq \frac{27R^2}{4r}$$

Proposed by Marin Chirciu-Romania

Solution by Mirsadix Muzefferov-Azerbaijan

$$\sum_{cyc} r_a \cdot \operatorname{ctg}^2 \left(\frac{A}{2} \right) = \sum_{cyc} r_a \cdot \left(\frac{p}{r_a} \right)^2 = p^2 \sum_{cyc} \frac{1}{r_a} = \frac{p^2}{r} \stackrel{\text{Mitrinovici}}{\leq} \frac{27R^2}{4r} \text{ (RHS)}$$

$$\sum_{cyc} r_a \cdot \operatorname{ctg}^2 \left(\frac{A}{2} \right) = \frac{p^2}{r} \stackrel{\text{Mitrinovici}}{\geq} \frac{27r^2}{r} = 27r \text{ (LHS)}$$

Equality holds for : $a = b = c$