ROMANIAN MATHEMATICAL MAGAZINE

In ΔABC the following relationship holds:

$$\frac{9r}{2R} \le \sum_{cyc} \frac{h_b.h_c}{bc} \le \frac{9}{4}$$

Proposed by Marin Chirciu-Romania

Solution by Mirsadix Muzefferov-Azerbaijan

$$\sum_{cyc} \frac{h_b.h_c}{bc} = \sum_{cyc} \frac{\frac{4F^2}{bc}}{bc} = 4F^2 \sum_{cyc} \left(\frac{1}{bc}\right)^2 \overset{Bergstrom}{\ge}$$

$$\ge 4F^2.\frac{\left(\sum_{cyc}\frac{1}{bc}\right)^2}{3} = 4F^2.\frac{\left(\frac{a+b+c}{abc}\right)^2}{3} = 4F^2.\frac{4p^2}{3(abc)^2} =$$

$$= \frac{16p^2F^2}{3.16F^2R^2} = \frac{p^2}{3R^2} \overset{Mitrinovici}{\ge} \frac{27Rr}{2.3R^2} = \frac{9r}{2R} \ (LHS)$$

$$\sum_{cyc} \frac{h_b.h_c}{bc} = 4F^2 \sum_{cyc} \left(\frac{1}{bc}\right)^2 = 4F^2.\frac{a^2+b^2+c^2}{a^2.b^2.c^2} \overset{Leibniz}{\le} 4F^2.\frac{9R^2}{16F^2R^2} = \frac{9}{4} \ (RHS)$$

$$Equality\ holds\ for: \quad a = b = c$$