ROMANIAN MATHEMATICAL MAGAZINE

In ΔABC the following relationship holds:

$$\sum_{cyc} h_a^2 \cdot \left(\frac{1}{h_b^3} + \frac{1}{h_c^3}\right) \geq \frac{2}{r}$$

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Solution by Mirsadix Muzefferov-Azerbaijan

$$\sum_{cyc} h_a^2 \cdot \left(\frac{1}{h_b^3} + \frac{1}{h_c^3}\right) = \sum_{cyc}\left(\frac{h_a^2}{h_b^3} + \frac{h_a^2}{h_c^3}\right) = \sum_{cyc}\left(\frac{\frac{4F^2}{a^2}}{\frac{8F^3}{b^3}} + \frac{\frac{4F^2}{a^2}}{\frac{8F^3}{c^3}}\right) =$$

$$= \frac{1}{2F}\sum_{cyc}\left(\frac{b^3}{a^2} + \frac{c^3}{a^2}\right) \overset{Radon}{\geq} \frac{1}{2F} \cdot \frac{\left(2(a+b+c)\right)^3}{\left(2(a+b+c)\right)^2} = \frac{1}{2F} \cdot 2(a+b+c) =$$

$$= \frac{a+b+c}{F} = \frac{2p}{pr} = \frac{2}{r}$$

$$Equality\ \ holds\ \ for:\ \ \ a = b = c$$