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In ΔABC the following relationship holds:

$$\sum_{cyc}\frac{h_bh_c}{bc}\le\sum_{cyc}\frac{r_br_c}{bc}$$

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$$RHS=\sum_{cyc}\frac{r_br_a}{bc}=4F^2\sum_{cyc}\frac{1}{bc(p-b)(p-c)}$$

$$LHS=\sum_{cyc}\frac{h_bh_c}{bc}=4F^2\sum_{cyc}\frac{1}{(bc)^2}$$

$$Let's\ prove\ \ that\ \ RHS-LHS\ge 0$$

$$4F^2\left(\sum_{cyc}\frac{1}{bc(p-b)(p-c)}-\frac{1}{(bc)^2}\right)=$$

$$=4F^2\left(\sum_{cyc}\frac{bc-(p-b)(p-c)}{b^2c^2(p-b)(p-c)}\right)=4F^2\sum_{cyc}\frac{bc-p^2+pb+pc-bc}{b^2c^2(p-b)(p-c)}=$$

$$=4F^2\sum_{cyc}\frac{p(b+c)-p^2}{b^2c^2(p-b)(p-c)}=4F^2\sum_{cyc}\frac{p(2p-a)-p^2}{b^2c^2(p-b)(p-c)}=$$

$$=4F^2\sum_{cyc}\frac{p(p-a)}{b^2c^2(p-b)(p-c)}\ge 0$$

$$Equality\ \ holds\ \ for:\ \ \ a=b=c$$