ROMANIAN MATHEMATICAL MAGAZINE

In ΔABC the following relationship holds:

$$\frac{3}{4r} \le \sum_{cyc} \frac{r_a}{bc} \le \frac{1}{r} - \frac{1}{2R}$$

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Solution by Mirsadix Muzefferov-Azerbaijan

$$\frac{r_a}{bc} = \frac{p.\tan\left(\frac{A}{2}\right)}{\frac{2F}{\sin(A)}} = \frac{p.\tan\left(\frac{A}{2}\right).\sin(A)}{2pr} = \frac{\sin\left(\frac{A}{2}\right).\sin(A)}{2r.\cos\left(\frac{A}{2}\right)} =$$

$$= \frac{2sin^2\left(\frac{A}{2}\right).\cos\left(\frac{A}{2}\right)}{2r.\cos\left(\frac{A}{2}\right)} = \frac{sin^2\left(\frac{A}{2}\right)}{r}$$

$$\sum_{cyc} \frac{sin^2\left(\frac{A}{2}\right)}{r} = \frac{1}{r}\left(\sum_{cyc} sin^2\left(\frac{A}{2}\right)\right) = \frac{1}{r}\left(1 - \frac{r}{2R}\right) = \frac{1}{r} - \frac{r}{2R} \quad (RHS)$$

$$\sum_{cyc} \frac{sin^2\left(\frac{A}{2}\right)}{r} = \frac{1}{r}\left(1 - \frac{r}{2R}\right) \ge \frac{1}{r}\left(1 - \frac{1}{4}\right) = \frac{3}{4r} \ (LHS)$$

$$Equality\ \ holds\ \ for: \quad a = b = c$$