

ROMANIAN MATHEMATICAL MAGAZINE

In $\triangle ABC$ the following relationship holds:

$$108r^3 \leq \sum_{cyc} a^2 h_a \leq 27R^2 r$$

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Solution by Mirsadix Muzefferov-Azerbaijan

$$\begin{aligned} \therefore \sum_{cyc} a^2 h_a &= \sum_{cyc} a^2 \cdot \frac{2F}{a} = 2F \sum_{cyc} a = 4pF = 4p^2 r \stackrel{\text{Mitrinovici}}{\lesseqgtr} 27R^2 r \\ \therefore \sum_{cyc} a^2 h_a &= 4p^2 r \stackrel{\text{Mitrinovici}}{\lesseqgtr} 4r \cdot 27r^2 = 108r^3 \end{aligned}$$

Equality holds for : $a = b = c$