ROMANIAN MATHEMATICAL MAGAZINE

In ΔABC te following relationship holds:

$$\sum_{cyc} \frac{r \cdot r_a}{r_b \cdot r_c} \leq \frac{3R}{2r} - 2$$

Proposed by Kostantinos Geronikolas-Greece

Solution by Mirsadix Muzefferov-Azerbaijan

$$r \cdot r_a = \frac{F}{p} \cdot \frac{F}{p-a} = \frac{F^2}{p(p-a)}; \qquad r_b \cdot r_c = p(p-a)$$

$$\sum_{cyc} \frac{r \cdot r_a}{r_b \cdot r_c} = \sum_{cyc} \frac{F^2}{p(p-a)} \cdot \frac{1}{p(p-a)} = \sum_{cyc} \frac{F^2}{p^2(p-a)^2} =$$

$$= F^2 \cdot \frac{(p-a)^2(p-b)^2 + (p-b)^2(p-c)^2 + (p-c)^2(p-a)^2}{p^2(p-a)^2(p-b)^2(p-c)^2} =$$

$$= F^2 \cdot \frac{r^2(4R+r)^2 - 2p^2r^2}{F^4} =$$

$$= \frac{r^2(4R+r)^2 - 2p^2r^2}{p^2r^2} = \frac{(4R+r)^2}{p^2} - 2 \overset{Euler}{\leq} \frac{81R^2}{4p^2} - 2 \leq$$

$$\leq^{(2p^2 \geq 27Rr)} \frac{81R^2}{2 \cdot 27Rr} - 2 = \frac{3R}{2r} - 2$$

$$Equality\ holds\ for\ a = b = c.$$