ROMANIAN MATHEMATICAL MAGAZINE

In ΔABC the following relationship holds:

$$\sum_{cyc}\sqrt{\frac{m_a}{h_a}}\cdot\sqrt{r_a}\le 3\sqrt{\frac{R}{2r}\left(\frac{R^2-Rr+r^2}{r}\right)}$$

Proposed by Kostantinos Geronikolas-Greece

Solution by Mirsadix Muzefferov-Azerbaijan

$$\sum_{cyc}\sqrt{\frac{m_a}{h_a}}\cdot\sqrt{r_a}\overset{CSB}{\overbrace{\le}}\sqrt{\sum_{cyc}\frac{m_a}{h_a}}\cdot\sqrt{\sum_{cyc}r_a}\overset{Panaitopol}{\overbrace{\le}}\sqrt{3\frac{R}{2r}}\cdot\sqrt{(4R+r)}=$$

$$=3\sqrt{\frac{R}{2r}\cdot\frac{4R+r}{3}}\le 3\sqrt{\frac{R}{2r}\left(\frac{R^2-Rr+r^2}{r}\right)}$$

$$Let's\ prove\ that:\ \frac{4R+r}{3}\le\frac{R^2-Rr+r^2}{r}$$

$$3R^2-7Rr+2r^2\ge 0$$

$$(R-2r)(3R-r)\ge 0\ \rightarrow\ R\ge 2r\ (Euler)$$

$$Equality\ holds\ for:\ \ a=b=c$$