ROMANIAN MATHEMATICAL MAGAZINE

In ΔABC the following relationship holds:

$$\left(\sum_{cyc}\cos(A)\right)\left(\sum_{cyc}sec^2(A)\right)\le\frac{3R}{r}$$

Proposed by Kostantinos Geronikolas-Greece

Solution by Mirsadix Muzefferov-Azerbaijan

$$\left(\sum_{cyc}\cos(A)\right)\left(\sum_{cyc}sec^2(A)\right)=\left(\sum_{cyc}\cos(A)\right)\left(\sum_{cyc}\frac{1}{cos^2\left(\frac{A}{2}\right)}\right)=$$

$$=\left(1+\frac{r}{R}\right)\left(1+\frac{(4R+r)^2}{p^2}\right)\le\left(1+\frac{r}{R}\right)\left(1+\frac{(4R+r)^2}{\frac{27Rr}{2}}\right)=$$

$$=\left(1+\frac{r}{R}\right)\left(1+\frac{32R^2+16Rr+2r^2}{27Rr}\right)=\left(1+\frac{r}{R}\right)\left(\frac{32\left(\frac{R}{r}\right)^2+43\left(\frac{R}{r}\right)+2}{27.\left(\frac{R}{r}\right)}\right)$$

$$Let\ \frac{R}{r}=x\ge 2\ \ Then\ \ let's\ \ prove\ \ it$$

$$\left(1+\frac{1}{x}\right)\left(\frac{32x^2+43x+2}{27x}\right)\le 3x$$

$$\frac{x+1}{x}.\frac{32x^2+43x+2}{27x}\le 3x$$

$$49x^3-75x^2-45x-2\ge 0$$

$$(x-2)(49x^2+23x+1)\ge 0$$

$$x-2\ge 0\rightarrow\ \ x\ge 2\ \ (Euler)\ (Proved)$$

$$Equality\ \ holds\ \ for:\ \ \ a=b=c$$