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In acute ΔABC the following relationship holds:

$$\frac{\sqrt{ctg(A)}}{a}+\frac{\sqrt{ctg(B)}}{b}+\frac{\sqrt{ctg(C)}}{c}\le\frac{\sqrt[4]{3}R}{4r^2}$$

Proposed by Kostantinos Geronikolas-Greece

Solution by Mirsadix Muzefferov-Azerbaijan

$$\sum_{cyc}\frac{\sqrt{ctg(A)}}{a}\overset{CSB}{\overbrace{\le}}\left(\sum_{cyc}\frac{1}{a^2}\right)^{\frac{1}{2}}\cdot\left(\sum_{cyc}ctg(A)\right)^{\frac{1}{2}}\overset{Steining}{\overbrace{\le}}$$

$$\frac{1}{2r}\cdot\sqrt{\frac{p^2-4Rr-r^2}{2pr}}\overset{Gerretsen}{\overbrace{\le}}\frac{1}{2r}\sqrt{\frac{4R^2+4Rr+3r^2-4Rr-r^2}{2pr}}=$$

$$\frac{1}{2r}\sqrt{\frac{2R^2+r^2}{pr}}\overset{Euler}{\overbrace{\le}}\frac{1}{2r}\cdot\frac{3R}{2}\cdot\sqrt{\frac{1}{pr}}\overset{Mitrinovici}{\overbrace{\le}}\frac{3R}{4r}\sqrt{\frac{1}{3\sqrt{3}r.r}}=\frac{\sqrt[4]{3}.R}{4r^2}$$

$$Equality\ \ holds\ \ for:\quad a=b=c$$