

ROMANIAN MATHEMATICAL MAGAZINE

In any ΔABC following relationship holds:

$$\frac{R}{r} \sum_{cyc} \sqrt{\frac{m_a^3}{w_a^2 h_a}} + \frac{1}{3r^2} \sum r_a r_b \geq \frac{7}{2} \sum \frac{r_a}{h_a} + \frac{9}{2} \sqrt{1 - \sum \left(\frac{m_a - w_a}{m_a + w_a} \right)^2}$$

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Solution by Tapas Das-India

$$\frac{R}{r} \sum_{cyc} \sqrt{\frac{m_a^3}{w_a^2 h_a}} \stackrel{m_a \geq w_a \geq h_a}{\geq} \frac{3R}{r} = 3x, \frac{R}{r} = x \geq 2 \text{ (Euler)}$$

$$\frac{1}{3r^2} \sum r_a r_b = \frac{1}{3} \left(\frac{s^2}{r^2} \right) \stackrel{\text{Gerretsn}}{\geq} \frac{1}{3} \frac{16Rr - 5r^2}{r^2} = \frac{1}{3} (16x - 5)$$

$$\frac{7}{2} \sum \frac{r_a}{h_a} = \frac{7}{2} \cdot \frac{1}{2} \sum \frac{a}{s-a} = \frac{7}{2} \left(\frac{2R}{r} - 1 \right) = \frac{7}{2} (2x - 1)$$

$$\frac{9}{2} \sqrt{1 - \sum \left(\frac{m_a - w_a}{m_a + w_a} \right)^2} \geq \frac{9}{2} \text{ as } m_a \geq w_a \text{ then } m_a - w_a \geq 0$$

We need to show:

$$\frac{R}{r} \sum_{cyc} \sqrt{\frac{m_a^3}{w_a^2 h_a}} + \frac{1}{3r^2} \sum r_a r_b \geq \frac{7}{2} \sum \frac{r_a}{h_a} + \frac{9}{2} \sqrt{1 - \sum \left(\frac{m_a - w_a}{m_a + w_a} \right)^2}$$

$$3x + \frac{1}{3} (16x - 5) \geq \frac{7}{2} (2x - 1) + \frac{9}{2}$$

$$18x + 32x - 10 \geq 42x - 21 + 27 \text{ or } 8x \geq 16 \text{ or } x \geq 2 \text{ True}$$

Equality holds for an equilateral triangle.