

ROMANIAN MATHEMATICAL MAGAZINE

In $\triangle ABC$ the following relationship holds:

$$\sum \left(\frac{m_a^{2026} + w_a^{2026} + g_a^{2026}}{3h_a^{2026}} \cdot \frac{r_b r_c}{r^2} \right) \geq 16 \frac{R}{r} - 5 \geq 13 \frac{R}{r} + 1 \geq 27$$

Proposed by Hikmat Mammadov-Azerbaijan

Solution by Tapas Das-India

We know that:

$h_a \leq g_a, h_a \leq m_a, h_a \leq w_a$ then:

$$\frac{m_a^{2026} + w_a^{2026} + g_a^{2026}}{3h_a^{2026}} = \frac{1}{3} \left(\left(\frac{m_a}{h_a} \right)^{2026} + \left(\frac{w_a}{h_a} \right)^{2026} + \left(\frac{g_a}{h_a} \right)^{2026} \right) \geq \frac{1}{3} (1 + 1 + 1) = 1$$

$$\begin{aligned} \sum \left(\frac{m_a^{2026} + w_a^{2026} + g_a^{2026}}{3h_a^{2026}} \cdot \frac{r_b r_c}{r^2} \right) &\geq \sum \left(\frac{r_b r_c}{r^2} \right) = \frac{s^2}{r^2} \stackrel{\text{Gerretsen}}{\geq} \\ &\geq \frac{16Rr - 5r^2}{r^2} = 16 \frac{R}{r} - 5 \end{aligned}$$

$$16 \frac{R}{r} - 5 = 13 \frac{R}{r} + \frac{3R}{r} - 5 \stackrel{\text{Euler}}{\geq} 13 \frac{R}{r} + 3 \times 2 - 5 = 13 \frac{R}{r} + 1 \stackrel{\text{Euler}}{\geq} 13 \times 2 + 1 = 27$$

Equality holds for an equilateral triangle.