ROMANIAN MATHEMATICAL MAGAZINE

Let ΔABC be equilateral and $CD \perp AB, D \in (AB)$ and P be any point on the inscribed circle of the triangle. Show that:

$$CD^2 < PA^2 + PB^2 + PC^2$$

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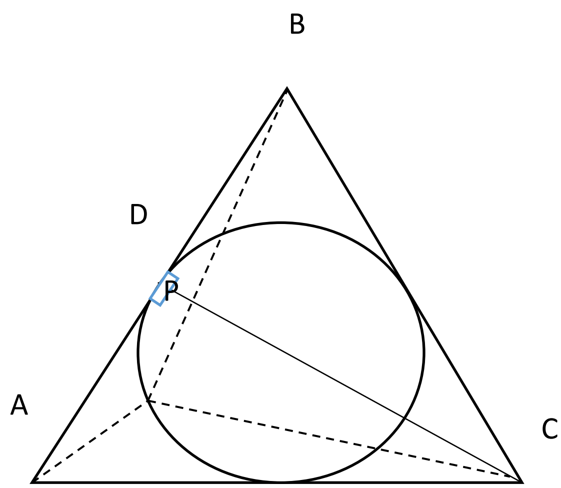
Solution by Mirsadix Muzefferov-Azerbaijan

Let in triangle ΔABC

$$AB = BC = AC = a \text{ and } CD = \frac{a\sqrt{3}}{2}$$



$$\text{In} \quad \left.\begin{array}{l} APB, AP + PB > a \\ APC, AP + PC > a \\ BPC, BP + PC > a \end{array}\right\} \Rightarrow AP + BP + CP > \frac{3a}{2}$$

$$3(PA^2 + PB^2 + PC^2) \geq (PA + PB + PC)^2 > \left(\frac{3a}{2}\right)^2 = \frac{9}{4}a^2 = \frac{9}{4} \cdot \frac{4CD^2}{3} = 3CD^2 \Rightarrow$$

$$\Rightarrow CD^2 < PA^2 + PB^2 + PC^2$$