

ROMANIAN MATHEMATICAL MAGAZINE

In any ΔABC , with $a \geq b \geq c, d \geq e \geq f \geq 0$ holds:

$$d(r_a - m_a) + e(r_b - m_b) + f(r_c - m_c) \geq 0$$

Proposed by Dang Ngoc Minh-Vietnam

Solution by Tapas Das-India

As $a \geq b \geq c$ then $r_a \geq r_b \geq r_c$ & $m_a \leq m_b \leq m_c$

$$d(r_a - m_a) + e(r_b - m_b) + f(r_c - m_c) =$$

$$= (dr_a + er_b + fr_c) - (dm_a + em_b + fm_c) \geq$$

Chebyshev

$$\stackrel{d \geq e \geq f}{\geq} \frac{1}{3}(d + e + f)(r_a + r_b + r_c) - \frac{1}{3}(d + e + f)(m_a + m_b + m_c) \geq$$

$$\stackrel{Leuenberger}{\geq} \frac{1}{3}(d + e + f)(4R + r) - \frac{1}{3}(d + e + f)(4R + r) = 0$$

Equality holds for $d=e=f=0$.