ROMANIAN MATHEMATICAL MAGAZINE

In ΔABC the following relationship holds:

$$m_a w_a < bc$$

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$$m_a^2 - \frac{(b+c)^2}{4} = \frac{2b^2+2c^2-a^2-b^2-c^2-2bc}{4} = \frac{(b-c)^2-a^2}{4}$$

$$= \frac{(b-c+a)(b-c-a)}{4} < 0 \text{ so } m_a < \frac{b+c}{2}$$

$$w_a = \frac{2bc}{b+c}\cos\frac{A}{2} \le \frac{2bc}{b+c} \text{ as } \cos\frac{A}{2} \le 1$$

$$\text{now } m_a w_a < \frac{2bc}{b+c}\cdot\frac{b+c}{2} = bc$$