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In $\triangle ABC$ the following relationship holds:

$$\sum_{cyc} \frac{a^4}{a+3c} \geq 18\sqrt{3}r^3$$

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Solution by Daniel Sitaru-Romania

$$\frac{a^4}{a+3c} + \frac{b^4}{b+3a} + \frac{c^4}{c+3b} \overset{\text{HOLDER}}{\geq} \frac{(a+b+c)^4}{9(a+3c+b+3a+c+3b)} =$$

$$= \frac{(2s)^4}{9 \cdot 8s} = \frac{2s^3}{9} \overset{MITRINOVICI}{\geq} \frac{2 \cdot \left(3\sqrt{3}r\right)^3}{9} = \frac{2 \cdot 27 \cdot 3\sqrt{3}r^3}{9} = 18\sqrt{3}r^3$$

Equality holds for $a = b = c$.