ROMANIAN MATHEMATICAL MAGAZINE

In any Δ ABC with P $\rightarrow$ an interior point,

the following relationship holds :

$$\sum_{\text{cyc}} \frac{PA^2}{a} \geq 2\sqrt{3}r$$

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Via Murray S. Klamkin (1975), if x, y, z are real numbers, then :

$$(x+y+z)(x.PA^2+y.PB^2+z.PC^2) \geq yza^2+zxb^2+xyc^2$$

and via $x \equiv \frac{1}{a}, y \equiv \frac{1}{b}, z \equiv \frac{1}{c}$, we get : $\left(\sum_{\text{cyc}} \frac{1}{a}\right)\left(\sum_{\text{cyc}} \frac{PA^2}{a}\right) \geq \sum_{\text{cyc}} \frac{a^2}{bc}$

$$\Rightarrow \sum_{\text{cyc}} \frac{PA^2}{a} \geq \frac{\sum_{\text{cyc}} a^3}{\sum_{\text{cyc}} ab} \overset{\text{Chebyshev}}{\geq} \frac{\left(\sum_{\text{cyc}} a^2\right)\left(\sum_{\text{cyc}} a\right)}{3\sum_{\text{cyc}} ab} \geq \frac{\left(\sum_{\text{cyc}} ab\right)\left(\sum_{\text{cyc}} a\right)}{3\sum_{\text{cyc}} ab}$$

$$\overset{\text{Mitrinovic}}{\geq} \frac{6\sqrt{3}r}{3} \text{ and so, : } \sum_{\text{cyc}} \frac{PA^2}{a} \geq 2\sqrt{3}r \ \ \forall \Delta \text{ ABC},$$

$'' = ''$ iff Δ ABC is equilateral (QED)