ROMANIAN MATHEMATICAL MAGAZINE

In any acute ΔABC the following relationship holds:

$$\frac{a\cot a}{h_a}+\frac{b\cot B}{h_b}+\frac{c\cot C}{h_c}=2$$

Proposed by Nguyen Hung Cuong – Vietnam

Solution by Daniel Sitaru – Romania

$$\frac{a\cot A}{h_a}+\frac{b\cot B}{h_b}+\frac{c\cot C}{h_c}=\sum_{cyc}\frac{a\cot A}{h_a}=\sum_{cy}\frac{a}{\frac{2F}{a}}\cdot\frac{\cos A}{\sin A}=\frac{1}{2F}\sum_{cyc}a^2\cdot\frac{b^2+c^2-a^2}{2bc\sin A}=$$

$$=\frac{1}{2F}\cdot\frac{1}{4F}\sum_{cyc}a^2(b^2+c^2-a^2)=$$

$$=\frac{1}{8F^2}(2a^2b^2+2b^2c^2+2c^2a^2-a^4-b^4-c^4)=\frac{1}{8F^2}\cdot 16F^2=2$$