

ROMANIAN MATHEMATICAL MAGAZINE

ΔABC with $\angle A = 90^\circ$, $D \in (AC)$, $E \in (BC)$, $AD = DE$,

$AB = BE$, DE perpendicular on BC &

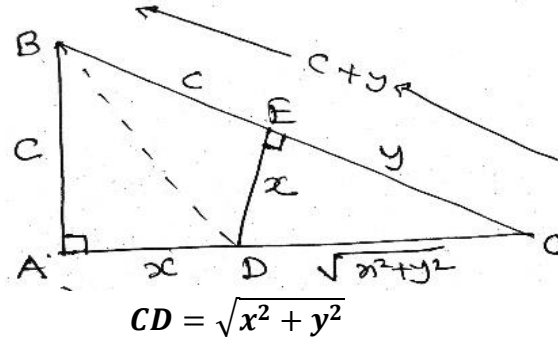
$\text{perimeter}(ABED) = \text{perimeter}(DCE)$. Prove that:

$$\frac{AB}{DE} = \text{golden ratio}$$

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Solution by Tapas Das-India

Let $AD = DE = x$ & $AB = BE = c$, $BC = c + y$, In ΔCDE , $\angle DEC = 90^\circ$ then



$$\text{perimeter}(ABED) = \text{perimeter}(DCE)$$

$$\text{or, } c + c + x + x = x + y + \sqrt{x^2 + y^2} \text{ or, } 2c + x - y = \sqrt{x^2 + y^2} \quad (1)$$

$$\begin{aligned} \Delta ABC \text{ we get } (c + y)^2 &= c^2 + \left(x + \sqrt{x^2 + y^2}\right)^2 \text{ or, } c^2 + 2cy + y^2 \\ &= c^2 + x^2 + x^2 + y^2 + 2x\sqrt{x^2 + y^2} \end{aligned}$$

$$cy = x^2 + x\sqrt{x^2 + y^2} \stackrel{(1)}{=} x^2 + x(2c + x - y) \text{ or } y(c + x) = 2x(c + x)$$

now from (1) we have :

$$2c + x - y = \sqrt{x^2 + y^2} \text{ or } 2c + x - 2x \stackrel{y=2x}{=} \sqrt{x^2 + (2x)^2} \text{ or, } 2c - x = x\sqrt{5} \text{ or}$$

$$\frac{c}{x} = \frac{\sqrt{5} + 1}{2}, \quad \text{so } \frac{AB}{DE} = \frac{c}{x} = \frac{\sqrt{5} + 1}{2} = \text{golden ratio} = \phi$$