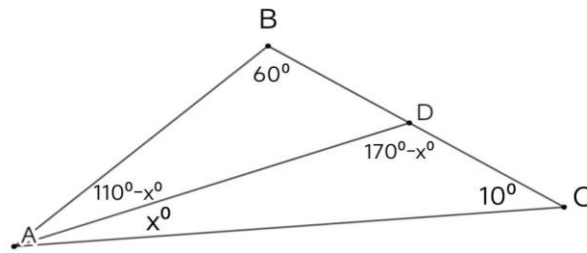


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In $\triangle ABC$, let $\angle B = 60^\circ$ and $\angle C = 10^\circ$. Point $D \in BC$ such that $|AC| = \sqrt{3} \cdot |BD|$, and $\angle DAC = x^\circ$, then find x .

Proposed by Adgozel Adgozalov-Azerbaijan



Solution by Mirsadix Muzefferov-Azerbaijan

$$\text{in } \triangle ABD \text{ sine rule } \frac{AD}{\sin 60^\circ} = \frac{BD}{\sin(110^\circ - x)}$$

$$\text{in } \triangle ADC \text{ sine rule } \frac{AD}{\sin 10^\circ} = \frac{AC}{\sin(170^\circ - x)}$$

$$\Rightarrow \begin{cases} \frac{2AD}{\sqrt{3}} = \frac{BD}{\cos(20^\circ - x)} \\ \frac{\sin 10^\circ}{AD} = \frac{\sin(10^\circ + x)}{AC} \end{cases} \Rightarrow 2 \cdot \sin 10^\circ \cdot \cos(20^\circ - x) = \sin(10^\circ + x)$$

$$\Rightarrow 2 \cdot \frac{1}{2} (\sin(30^\circ - x) + \sin(x - 10^\circ)) = \sin(x + 10^\circ)$$

$$\Rightarrow \sin(30^\circ - x) = \sin(x + 10^\circ) - \sin(x - 10^\circ) \Rightarrow \sin(30^\circ - x) = 2\sin 10^\circ \cdot \cos x$$

$$\Rightarrow \sin 30^\circ \cdot \cos x - \cos 30^\circ \cdot \sin x = 2\sin 10^\circ \cdot \cos x \Rightarrow \frac{\cos x}{2} - \frac{\sqrt{3}}{2} \sin x = 2\sin 10^\circ \cdot \cos x$$

$$\Rightarrow \tan x = \frac{1 - 4\sin 10^\circ}{\sqrt{3}} \quad \cdot \text{ let's prove that : } 1 - 4\sin 10^\circ = \sqrt{3}\tan 10^\circ$$

$$(1 - 4\sin 10^\circ) \cdot \cos 10^\circ = \sqrt{3}\sin 10^\circ \Rightarrow \cos 10^\circ - 2\sin 20^\circ = \sqrt{3}\sin 10^\circ$$

$$\Rightarrow \cos 10^\circ - \sqrt{3}\sin 10^\circ = 2\sin 20^\circ \Rightarrow \frac{1}{2} \cdot \cos 10^\circ - \frac{\sqrt{3}}{2} \sin 10^\circ = \sin 20^\circ$$

ROMANIAN MATHEMATICAL MAGAZINE

$$\Rightarrow \sin 30^\circ \cdot \cos 10^\circ - \cos 30^\circ \cdot \sin 10^\circ = \sin 20^\circ \Rightarrow \sin(30^\circ - 10^\circ) = \sin 20^\circ$$

$$\Rightarrow \sin 20^\circ = \sin 20^\circ \quad (\text{true}). \text{ So } x = 10^\circ$$