ROMANIAN MATHEMATICAL MAGAZINE

If in $\Delta ABC, = 80°, B = C = 50°$, $AB = AC = a$, $BC = b$ then:

$$3a^2b = b^3 + a^3\sqrt{3}$$

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Solution by Tapas Das-India

$$\frac{b}{\sin 80°} = \frac{a}{\sin 50°} \Rightarrow \frac{b}{\sin 80°} = \frac{a}{\sin(90° - 40°)}$$
$$\Rightarrow \frac{b}{2sin40°cos40°} = \frac{a}{\cos 40°} \Rightarrow b = 2asin40°$$

$$3a^2b = 3a^2 \times 2asin40° = 6a^3sin40°$$

$$b^3 + a^3\sqrt{3} = 8a^3 \sin^3 40° + a^3\sqrt{3}$$

$$We\ need\ to\ show:$$

$$6a^3sin40° = 8a^3 \sin^3 40° + a^3\sqrt{3}or, 6sin40° - 8\sin^3 40° = \sqrt{3}$$

$$L.H.S = 6sin40° - 8\sin^3 40° = 2(3sin40° - 4\sin^3 40°) =$$
$$= 2\sin 3.40° = 2\sin 120° = 2 \times \frac{\sqrt{3}}{2} = \sqrt{3}$$