ROMANIAN MATHEMATICAL MAGAZINE

In ΔABC , let $M \in AC, K \in BC$,
$N \in AB$ where M is the midpoint of AC , then:

$$[MNK]_{\min.} = \frac{6abc - ba^2 - bc^2}{64R}$$

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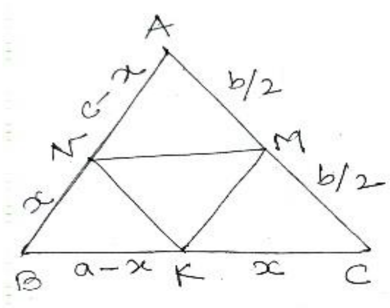

$$Let\ BN = KC = x \ \&\ 0 \le x < \min(a, c),$$

$$[AMN] = \frac{1}{2}\left(\frac{b}{2}\right).(c - x)\sin A = \frac{c-x}{2c}[ABC] \text{as}$$

$$\left([ABC] = \frac{1}{2}bc\sin A\ \ then \sin A = \frac{2}{bc}[ABC]\right)$$

$$similarly, [CMK] = \frac{1}{2}.\frac{b}{2}.xsinC = \frac{x}{2a}[ABC] \& [BNK] = \frac{1}{2}x(x - a)[ABC]$$

$$Now, [MNK] = [ABC] - [AMN] - [BNK] - [CMK] or, \frac{[MNK]}{[ABC]}$$

$$= 1 - \frac{c - x}{2c} - \frac{x(a - x)}{ac} - \frac{x}{2a} = \frac{1}{2ac}(2x^2 - (a + c)x + ac)\ (1)$$

$$let\ f(x) = 2x^2 - (a + c)x + ac\ , f'(x) = 4x - (a + c), f''(x) = 4 > 0\ , f'(x)$$

$$= 0\ gives\ x = \frac{a + c}{4}\ and\ f'\left(\frac{a + c}{4}\right) > 0$$

$$so\ f(x) minimum\ at\ x = \frac{a + c}{4} \ \&\ f\left(\frac{a + c}{4}\right) = \frac{2(a + c)^2}{16} - \frac{(a + c)^2}{4} + ac$$

$$= \frac{6ac - a^2 - c^2}{8}$$

$$From (1) we\ get$$

$$[MNK]_{\min.} = \frac{6ac - a^2 - c^2}{8 \times 2ac}.[ABC] = \frac{6ac - a^2 - c^2}{8 \times 2ac}.\frac{abc}{4R} = \frac{6abc - ba^2 - bc^2}{64R}$$