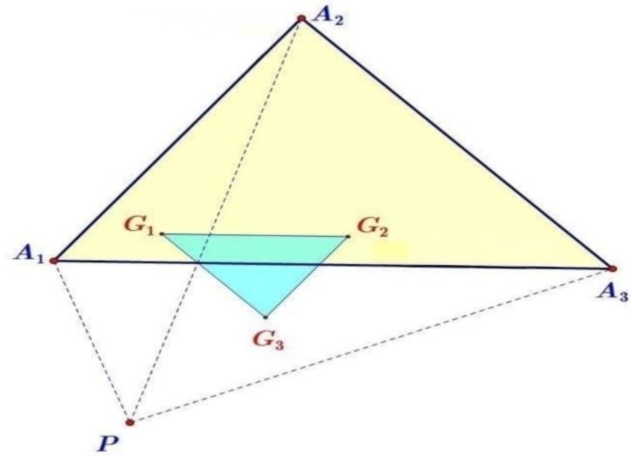
ROMANIAN MATHEMATICAL MAGAZINE



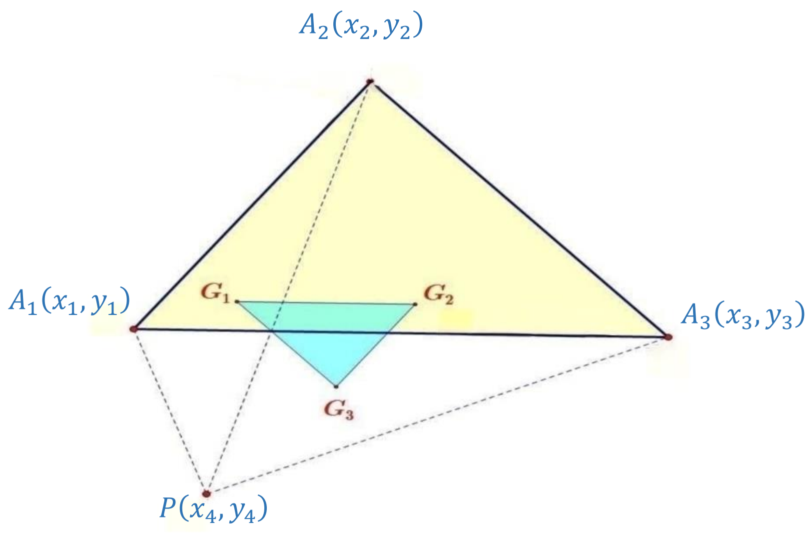
Let P, A_1, A_2, A_3 *are any points on a plane and* G_1, G_2, G_3 *are the centroids of* $PA_1A_2, PA_2A_3, PA_3A_1$ **respectively.** *Prove that* :

1) $G_1G_2 \parallel A_1A_3, G_2G_3 \parallel A_1A_2$ *and* $G_3G_1 \parallel A_2A_3$

2) $[\Delta A_1A_2A_3] = 9[\Delta G_1G_2G_3]$

Proposed by Jafar Nikpour-Iran

Solution by Mirsadix Muzefferov-Azerbaijan



$$\text{Let's determine the coordinates of the centers of gravity } G_1, G_2, G_3$$

$$G_1\left(\frac{x_1+x_2+x_4}{3};\frac{y_1+y_2+y_4}{3}\right), G_2\left(\frac{x_2+x_3+x_4}{3};\frac{y_2+y_3+y_4}{3}\right), G_3\left(\frac{x_1+x_3+x_4}{3};\frac{y_1+y_3+y_4}{3}\right)$$

$$\overrightarrow{G_1G_2}=\left(\frac{x_3-x_1}{3};\frac{y_3-y_1}{3}\right), \overrightarrow{A_1A_3}=(x_3-x_1;y_3-y_1) \Rightarrow G_1G_2 \parallel A_1A_3$$

$$\overrightarrow{G_2G_3}=\left(\frac{x_1-x_2}{3};\frac{y_1-y_2}{3}\right), \overrightarrow{A_1A_2}=(x_2-x_1;y_2-y_1) \Rightarrow G_2G_3 \parallel A_1A_2$$

$$\overrightarrow{G_3G_1}=\left(\frac{x_2-x_3}{3};\frac{y_2-y_3}{3}\right), \overrightarrow{A_2A_3}=(x_3-x_2;y_3-y_2) \Rightarrow G_3G_1 \parallel A_2A_3$$

$$\left.\begin{array}{l} G_1G_2 \parallel A_1A_3 \\ G_2G_3 \parallel A_1A_2 \\ G_3G_1 \parallel A_2A_3 \end{array}\right\} \Rightarrow \Delta G_1G_2G_3 \sim \Delta A_1A_2A_3 \Rightarrow k=3 \text{ here } k- \text{ is the ratio of similitude}$$

$$\text{Then } [\Delta A_1A_2A_3] \div [\Delta G_1G_2G_3] = k^2 = 9.$$