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$$If\ a,b,c>0\ then:$$

$$\left(\sum a\sqrt{\frac{b}{a+b}}\right)\left(\sum b\sqrt{\frac{b}{a+b}}\right)\le\frac{3}{2}(ab+bc+ca)$$

Proposed by Nguyen Hung Cuong-Vietnam

Solution by Tapas Das-India

$$\left(\sum a\sqrt{\frac{b}{a+b}}\right)=\sum\sqrt{ab}\sqrt{\frac{a}{a+b}}\overset{C-S}{\le}\sqrt{\left(\sum ab\right)\left(\sum\frac{a}{a+b}\right)}$$

$$similarly,\left(\sum b\sqrt{\frac{a}{a+b}}\right)=\sum\sqrt{ab}\sqrt{\frac{b}{a+b}}\overset{C-S}{\le}\sqrt{\left(\sum ab\right)\left(\sum\frac{b}{a+b}\right)}$$

$$now,\left(\sum a\sqrt{\frac{b}{a+b}}\right)\left(\sum b\sqrt{\frac{b}{a+b}}\right)\le\left(\sqrt{\left(\sum ab\right)\left(\sum\frac{a}{a+b}\right)}\right)\left(\sqrt{\left(\sum ab\right)\left(\sum\frac{b}{a+b}\right)}\right)$$

$$=(ab+bc+ca)\sqrt{\left(\sum\frac{a}{a+b}\right)\left(\sum\frac{b}{a+b}\right)}\overset{AM-GM}{\le}(ab+bc+ca)\frac{\sum\frac{a}{a+b}+\sum\frac{b}{a+b}}{2}=$$

$$=(ab+bc+ca)\frac{1}{2}\sum\left(\frac{a}{a+b}+\frac{b}{b+a}\right)=\frac{3}{2}(ab+bc+ca)$$

Equality holds for a=b=c