ROMANIAN MATHEMATICAL MAGAZINE

$$If\ a,b,c>2021,\frac{1}{a}+\frac{1}{b}+\frac{1}{c}=\frac{2}{2021}\ then:$$

$$\sqrt{a+b+c}\geq\sqrt{a-2021}+\sqrt{b-2021}+\sqrt{c-2021}$$

Proposed by Nguyen Hung Cuong-Vietnam

Solution by Tapas Das-India

$$Let\ k=2021\ then\ \frac{1}{a}+\frac{1}{b}+\frac{1}{c}=\frac{2}{k}$$

$$R.H.S=\sqrt{a-2021}+\sqrt{b-2021}+\sqrt{c-2021}=\sqrt{a-k}+\sqrt{b-k}+\sqrt{c-k}=$$

$$=\sqrt{\frac{a-k}{a}.a}+\sqrt{\frac{b-k}{b}.b}+\sqrt{\frac{c-k}{c}.c}=\sqrt{\left(1-\frac{k}{a}\right).a}+\sqrt{\left(1-\frac{k}{b}\right).b}+\sqrt{\left(1-\frac{k}{c}\right).c}\leq$$

$$\overset{C-S}{\leq}\sqrt{\left(1-\frac{k}{a}+1-\frac{k}{b}+1-\frac{k}{c}\right)(a+b+c)}=\sqrt{\left(3-\left(\frac{k}{a}+\frac{k}{b}+\frac{k}{c}\right)\right)(a+b+c)}=$$

$$=\sqrt{\left(3-k.\frac{2}{k}\right)(a+b+c)}=\sqrt{a+b+c}$$

$$Equality\ holds\ for\ a=b=c=\frac{3k}{2}=\frac{3}{2}\times 2021$$