ROMANIAN MATHEMATICAL MAGAZINE

$$If\ a,b,c>0, abc+ab+bc+ca=4\ then:$$

$$\frac{1}{a+2b+6}+\frac{1}{a+2b+6}+\frac{1}{a+2b+6}\le\frac{1}{3}$$

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Solution by Tapas Das-India

$$abc+ab+bc+ca=4\ can\ be\ written\ as\ \frac{1}{a+2}+\frac{1}{b+2}+\frac{1}{c+2}=1$$

$$\frac{1}{a+2b+6}=\frac{1}{(a+2)+(b+2)+(b+2)}\overset{AM-HM}{\le}\frac{1}{9}\left(\frac{1}{a+2}+\frac{1}{b+2}+\frac{1}{b+2}\right)$$

$$\frac{1}{a+2b+6}+\frac{1}{a+2b+6}+\frac{1}{a+2b+6}=\sum\frac{1}{a+2b+6}\le$$

$$\le\frac{1}{9}\sum\left(\frac{1}{a+2}+\frac{1}{b+2}+\frac{1}{b+2}\right)=\frac{1}{3}\sum\frac{1}{a+2}=\frac{1}{3}$$

Equality holds for a=b=c=1.