ROMANIAN MATHEMATICAL MAGAZINE

$$If\ a,b,c,d>0\ and \frac{1}{1+a^4}+\frac{1}{1+b^4}+\frac{1}{1+c^4}+\frac{1}{1+d^4}=1$$

$$then\ prove\ that\ abcd\geq 3$$

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Solution by Tapas Das-India

$$Let\ x=\frac{1}{1+a^4},y=\frac{1}{1+b^4},z=\frac{1}{1+c^4},w=\frac{1}{1+d^4}\ then\ x+y+z+w=1$$

$$1+a^4=\frac{1}{x}\ or, a^4=\frac{1}{x}-1=\frac{1-x}{x}\overset{x+y+z+w=1}{=}\frac{y+z+w}{x}\overset{AM-GM}{\geq}\frac{3\sqrt[3]{yzw}}{x}$$

$$similarly, b^4\geq\frac{3\sqrt[3]{xzw}}{y},c^4\geq\frac{3\sqrt[3]{yxw}}{z},d^4\geq\frac{3\sqrt[3]{yzx}}{w}$$

$$Now\ a^4b^4c^4d^4\geq 81 or, abcd\geq 3$$

$$Equality\ holds\ for\ a=b=c=d=\sqrt[4]{3}$$