ROMANIAN MATHEMATICAL MAGAZINE

$$If\ a,b,c>0\ and\ a+b+c=3\ then:$$

$$a^4b+b^4c+c^4a+18\geq 7(ab+bc+ca)$$

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Solution by Tapas Das-India

$$x^4+1+1+1\overset{AM-GM}{\geq}4x\ or, x^4\geq 4x-3\ (1)$$

$$(a+b+c)^2\geq 3(ab+b+ca) or, ab+bc+ca\overset{a+b+c=3}{\leq}3\ (2)$$

$$a^4b+b^4c+c^4a+18\overset{(1)}{\geq}4(ab+bc+ca)-9+18=4(ab+bc+ca)+9$$

$$\geq 4(ab+bc+ca)+3(ab+bc+ca)=7(ab+bc+ca)$$

$$Equality\ holds\ for\ a=b=c=1$$