ROMANIAN MATHEMATICAL MAGAZINE

$$If\ a,b,c>2, abc=2(a+b+c)\ then$$

$$(a-2)(b-2)(c-2)\le 18\sqrt{6}-44$$

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$$Let\ x=a-2, y=b-2, z=c-2\ , a,b,ç>2\ then\ x,y,z>0$$

$$abc=2(a+b+c)\ or, (x+2)(y+2)(z+2)=2(x+y+z+6)$$

$$or, xyz+2(xy+yz+zx)+4(x+y+z)+8=2(x+y+z)+12$$

$$or, xyz+2\times 3(xyz)^{\frac{2}{3}}+2\times 3(xyz)^{\frac{1}{3}} \overset{AM-GM}{\le} 4\ or, t^3+6t^2+6t \overset{xyz=t^3}{\le} 4$$

$$or, (t+2)(t^2+4t-2)\le 0\ now, t^3=xyz>0\ then\ t+2>0\ which\ indicate\ that$$

$$(t^2+4t-2)\le 0\ or, (t+2)^2\le 6\ or, t+2\le\sqrt{6}\ or, t\le\sqrt{6}-2$$

$$(a-2)(b-2)(c-2)=xyz=t^3\le\left(\sqrt{6}-2\right)^3=18\sqrt{6}-44$$

$$Equality\ holds\ for\ a=b=c=\sqrt{6}$$