ROMANIAN MATHEMATICAL MAGAZINE

$$If\ a,b,c>0, a+b+c=3\ then\ prove\ that:$$

$$3(a^2+b^2+c^2)+\frac{ab+bc+ca}{a^2+b^2+c^2}\geq 10$$

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$$m=a^2+b^2+c^2=\frac{a^2}{1}+\frac{b^2}{1}+\frac{c^2}{1}\overset{Radon}{\geq}\frac{(a+b+c)^2}{3}\overset{a+b+c=3}{=}3$$

$$(a^2+b^2+c^2)+2(ab+bc+ca)=(a+b+c)^2\overset{a+b+c=3}{=}9$$

$$or,(ab+bc+ca)=\frac{9-(a^2+b^2+c^2)}{2}$$

$$now\ L.H.S=3(a^2+b^2+c^2)+\frac{9-(a^2+b^2+c^2)}{2(a^2+b^2+c^2)}\overset{m=a^2+b^2+c^2}{=}3m+\frac{9-m}{2m}$$

$$We\ need\ to\ show: 3m+\frac{9-m}{2m}\geq 10$$

$$6m^2-21m+9\geq 0\ or\ 3(m-3)(2m-1)\geq 0$$

$$True\ as\ m\geq 3\ , Equality\ holds\ for\ a=b=c=1$$