

ROMANIAN MATHEMATICAL MAGAZINE

If $a, b > 0$; $ab = 1$ then:

$$\frac{a}{b^3 + 3a} + \frac{b}{a^3 + 3b} \leq \frac{1}{2}$$

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$$\frac{a}{b^3 + 3a} + \frac{b}{a^3 + 3b} \leq \frac{1}{2}$$

$$2a(a^3 + 3b) + 2b(b^3 + 3a) \leq (b^3 + 3a)(a^3 + 3b)$$

$$2a^4 + 6ab + 2b^4 + 6ab \leq (ab)^3 + 3b^4 + 3a^4 + 9ab$$

$$2(a^4 + b^4) + 6 + 6 \leq 1 + 3(a^4 + b^4) + 9$$

$$a^4 + b^4 \geq 12 - 10 = 2$$

$$a^4 + b^4 \geq 2 \quad (\text{to prove})$$

$$a^4 + b^4 \stackrel{AM-GM}{\geq} 2\sqrt{(ab)^4} = 2\sqrt{1^4} = 2 \cdot 1 = 2$$

Equality holds for $a = b = 1$.