ROMANIAN MATHEMATICAL MAGAZINE

$$\boldsymbol{If\ \ a,b,c>0\ \ then\ \ prove\ \ that:}$$

$$\sum_{cyc}\frac{a^2b^2}{a+b}\le\sum_{cyc}\frac{b^2c^2}{a+b}$$

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Solution by Mirsadix Muzefferov-Azerbaijan

$$\frac{a^2b^2}{a+b}+\frac{c^2b^2}{c+b}+\frac{a^2c^2}{a+c}\le\frac{c^2b^2}{a+b}+\frac{a^2b^2}{a+c}+\frac{a^2c^2}{b+c}$$

$$Wlog:a\le b\le c\ \ ,a^2b^2\le a^2c^2\le c^2b^2$$

$$a,b,c>0,\quad \begin{cases}a+b\le a+c\le b+c\\ \dfrac{1}{a+b}\ge\dfrac{1}{a+c}\ge\dfrac{1}{b+c}\end{cases}$$

$$According\ \ to\ \ the\ \ conditions\ \ of\ \ the\ \ Rearrangement\ inequality:$$

$$B_{min}=\frac{a^2b^2}{a+b}+\frac{c^2b^2}{c+b}+\frac{a^2c^2}{a+c},\qquad C_{mixed}=\frac{c^2b^2}{a+b}+\frac{a^2b^2}{a+c}+\frac{a^2c^2}{b+c}$$

$$So,\ \ B_{min}\le C_{mixed}.\quad Equality\ \ holds\ \ for:\quad a=b=c$$